Business Jet Large-Scale Model Approximation and Vibration Control

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Abstract: In this paper, a procedure allowing to design an H_∞-based low order anti-vibration controller is proposed. The contributions of the paper lie in the first specification and application of recent *(i)* model approximation and *(ii)* H_∞ structured controller tuning techniques on a complex aeroelastical aircraft model, used by engineers to design control strategies. The entire procedure - *i.e.* approximation and control - is assessed on a Business Jet large-scale generic model, highlighting the effectiveness of the approach in a concrete use-case.

Keywords: Aircraft modelling, Model approximation, Vibration control, Structured H_∞ control

1. INTRODUCTION

1.1 Motivations and industrial framework

Aircraft control engineers have often to cope with many complex technical and practical problems such as aeroservoelastic control based on dedicated models. Aeroservoelastic models take into account the physics involved in the aeroelastic phenomenon (structural loads, unsteady aerodynamic loads) and the flight control system behavior (actuator dynamics, sensors bandwidth, measurement delays). These state-space models *(i)* have a large scale (order n around 1000). Uncertainty parameters are applied to these models, accounting for variabilities and uncertainties on aircraft characteristics (*e.g.* mass, flight altitude and speed). Aeroservoelastic models usually have perturbation inputs and control inputs (aerodynamic control surfaces), and sensors (*e.g.* accelerometers and gyrometers) outputs.

Based on these models, nominal flight control laws *(ii)* are designed with the aim to ensure good handling qualities to the aircraft (fast, smooth and accurate response to pilot commands) and a rejection of the aeroelastic modes on control. However some aerodynamic perturbations are likely to occur and could generate an undesirable response of some medium frequency modes (vibrations) on some parts of the aircraft within a specific frequency range (*e.g.* perturbations on the rear part of the aircraft (A/C) could generate vibrations in the cockpit or in the cabin).

The vibration control aims at attenuating the encountered vibrations by the deflection of control surfaces (*e.g.* elevators located on the rear part of the A/C). This deflection is calculated through specific control laws using information from sensors such as accelerometers and gyrometers. These control laws are subjected to three main constraints: reduce the vibrations, generate negligible effects on the handling qualities of the aircraft and reduce as much as possible effects on structural modes.

This paper reports results obtained within the joint collaboration between Onera and Dassault-Aviation on the development of advanced methodologies for vibration control design applied to a business jet (or BizJet) aircraft generic model. More specifically, and with reference to Figure 1, the paper is attached to items *(i)* and *(ii)*, *i.e.* given a large-scale BizJet model and its nominal flight control system (blue elements), approximate it and design an additional dedicated anti-vibration control law that must be added in the original loop (red block).

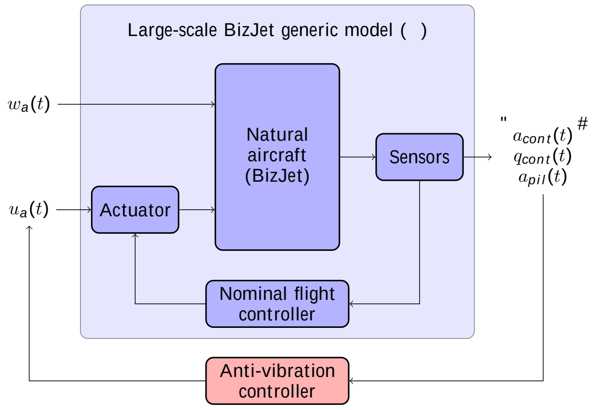

Fig. 1. BizJet model ( ) and Anti-vibration controller.

The design of dedicated anti-vibration controller, in the Dassault-Aviation industrial framework including large-scale dynamical systems, is a challenging problem:

- the large number of states involved in the dynamical model (source of computation complexity),
- the undesirable vibrations should be attenuated over a specific frequency range (linked to passenger and pilot comfort specifications),
- the nominal flight control law performances must be preserved and high frequency flexible - lightly - damped modes must remain sufficiently stable,
- the anti-vibration control is based on the deflection of existing control surfaces.

1.2 Paper contributions and structure

Within this framework, the contribution of this paper consists in describing a dedicated methodology to design an anti-vibration controller for the BizJet application. The approach consists in a three steps procedure: first by *(i)* reducing the system dimension using (frequency-limited) H_2 optimal model approximation methods, then *(ii)* in writing the generalized standard scheme using specific weighting filters that capture frequency-oriented constraints and objectives. Finally *(iii)*, a structured controller exploiting recent H_∞ non-smooth optimization tools is designed (see *e.g.* Burke et al. (2006); Apkarian and Noll (2006)). The entire procedure is tested in numerical simulations involving the full large-scale generic BizJet LTI model.

The rest of the paper is organized as follows. First, Section 2 describes the considered model and formalizes the performance objectives. Illustration of recent H_2-oriented model approximation techniques are also provided[1]. Section 3 describes the main contribution of this work, namely, the design of a dedicated anti-vibration control law, achieved in the H_∞ framework. Simulations on the complete large-scale aeroelastic generic aircraft model are performed as well, assessing the effectiveness of the approach. A discussion on the achieved performances with respect to control design parameters is also carried out. Finally, conclusions and future studies are drawn in Section 4.

Throughout the paper, the following notations will be used: (*resp.* $H(s)$) denotes the full order state-space model (*resp.* transfer function) of order n and $\hat{}$ (*resp.* $\hat{H}(s)$) stands for the reduced order state-space model (*resp.* transfer function) of order $r \quad n$. Given the matrices $M = \begin{matrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{matrix}$ and K of suitable dimensions, $F_l(M,K)_{w\rightarrow z} = M_{11} + M_{12}K(I \quad M_{22}K)^{-1}M_{21}$ denotes the transfer from w to z of the lower linear fractional transformation operator that interconnects M with K. I_n denotes the identity matrix of dimension n, $Re(z)$ the real part of the complex number z. $\lambda(.)$ and $\mathrm{tr}(.)$ stand for the eigenvalue and trace operators, respectively.

2. BIZJET AIRCRAFT MODELLING, APPROXIMATION AND PROBLEM FORMULATION

2.1 Large-scale modelling

The considered dynamical aircraft model, provided by Dassault Aviation, represents a longitudinal BizJet generic aircraft at a given flight condition (mass, flight altitude and speed). This aeroservoelastic model form includes the aeroelastic model coupled to the flight control system (see Figure 1). This system can be represented in its state-space or transfer function $H(s)$ form as:

$$: \quad \begin{matrix} \underline{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \end{matrix} \quad \text{or } H(s) = C(sI_n \quad A)^{-1}B, \tag{1}$$

where $A\ 2\ \mathbb{R}^{n\times n}$, $B\ 2\ \mathbb{R}^{n\times n_u}$ and $C\ 2\ \mathbb{R}^{n_y\times n}$ (with n 600, $n_u = 2$ and $n_y = 3$ are the number of states, inputs and outputs, respectively). In the considered application, the input vector is composed of

$w_a(t)$, the external disturbance input representing the aerodynamic perturbation located on the rear part of the A/C,
$u_a(t)$, anti-vibration control input (elevator on the rear part of the A/C),

and the output vector is composed of

$a_{cont}(t)$, the vertical acceleration output, used by the nominal flight controller,
$q_{cont}(t)$, the pitch rate of the aircraft, used by the nominal flight controller,
$a_{pil}(t)$, the acceleration sensor located close to the pilot's seat, which has to be monitored for comfort issues.

2.2 H_2 model approximation

As the original system is of large-scale, the application of the control optimization tools (either LMI, Riccati or non-smooth) oftena1c]TJ07383.8F9626 Tf. [(crate)-44 9.9626 Tf

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[1] As model approximation is not the main topic of the paper, for additional details, reader should refer to Poussot-Vassal (2011) and Vuillemin et al. (2013b).

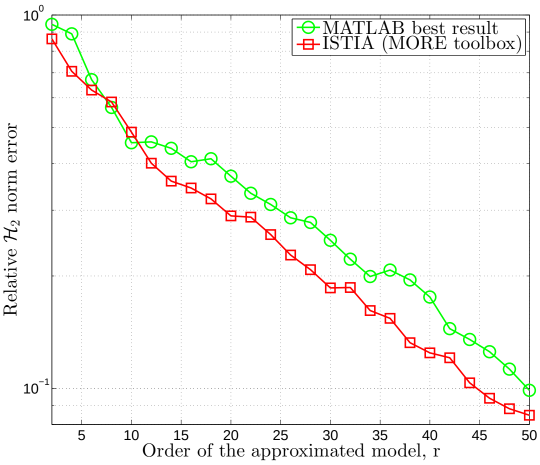

Fig. 2. Mismatch error as a function of the order r of the approximated model for: BT (green rounded line) and ISTIA of the MORE toolbox (red squared line).

Objectives: The anti-vibration controller objectives are:

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wherer

- $u(t) = u_a(t)$, representing the only control input,
- the measured output vector $y(t)$ is composed of $a_{cont}(t)$, $q_{cont}(t)$ and $a_{pil}(t)$.
- $z(t) = W_o z(t)$, which gathers the output performances. This signal represents the weighted output performances and is obtained by filtering $z(t)$, the considered performance outputs of ^ (see below).

More specifically, the performance signal $z(t)$ is linked, through relevant filters, to signals $u_a(t)$, $a_{pil}(t)$ and the error $a^*_{cont}(t) \quad a_{cont}(t)$, where $a^*_{cont}(t)$ can be viewed as a reference signal allowing to fulfil the first item of the constraint set (more details are given below in the W_e filter description). The performance output is given by:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \underbrace{\begin{bmatrix} W_e & 0 & 0 \\ 0 & W_u & 0 \\ 0 & 0 & W_{av} \end{bmatrix}}_{W_o} \begin{bmatrix} a^*_{cont} \quad a_{cont} \\ u \\ a_{pil} \end{bmatrix} \qquad (8)$$

where

- the performance output z_1, is given by $z_1 = W_e(a^*_{cont} \quad a_{cont})$ with W_e a generic nth order low pass filter

$$W_e = G_{!_c} \frac{1}{G_{le}} \left(\frac{\frac{1}{w_{e2}} s + 1}{\frac{1}{w_{e1}} s + 1} \right)^{n} \qquad (9)$$

where w_{e1}, G_{le}

otherwise, the gains of weighting filters are modified and return step (2).

Remark 4. (H_∞ optimisation for H_2 objectives). In Problem 1, objectives and constraints are formulated through H_2 metrics, although an H_∞ problem is solved. We emphasize that by minimizing the influence of perturbations on the selected output signals, a satisfactory confidence level regarding the minimisation of energy included in criteria J_1 and J_2 is expected. Moreover, to the best of authors' knowledge, there still does not exist numerical tool that allows the minimization of a H_2 criteria on a frequency range. Moreover, as illustrated in the next subsection, a consistent generalized plant formulation linked with the H_∞ objective provides good performances in J_1 and J_2 as well (Figure 5). Finally, in view of robust design extensions, the H_∞ framework fits better.

3.3 Numerical simulation results

Figure 5 illustrates the evolution of the H_∞ norm, and improvement ratios on criteria J_1 ($i.e.$ $\frac{\mathcal{J}_1^{nom}-\mathcal{J}_1}{\mathcal{J}_1^{nom}}$) and J_2 as functions of the order n_c of the designed controller, where J_1^{nom} and J_2^{nom} denote the nominal performances ($i.e.$ performances without any anti-vibration law).

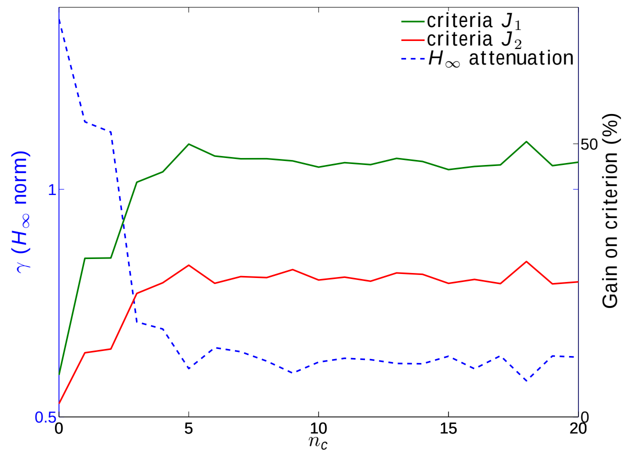

Fig. 5. Optimisation criterion - H_∞ norm - (dotted blue line), improvement ratios on J_1 and J_2 (continuous red and green lines, respectively).

As expected, when the H_∞ norm - solution of the optimization problem - decreases the corresponding improvement on criteria J_1 and J_2 increase (in %). As non-smooth optimization provides locally optimal controllers, one may note that the H_∞ norm decreases in a non-monotonic way. Nevertheless, a reduction ratio of nearly 50% is reached with controllers of a greater order than n_c = 5, with little reduction benefit brought by higher orders, as illustrated on Figure 5. Figure 5 also highlights the fact the chosen H_∞ objective is consistent with the H_2 objectives mentioned in Problem 1. Indeed, as γ - the closed-loop H_∞ norm - decreases, J_1 is minimized ($i.e.$ $\frac{\mathcal{J}_1^{nom}-\mathcal{J}_1}{\mathcal{J}_1^{nom}}$ increases) with a similar profile. This point confirms the fact that the generalized plant has been well defined. So it is for the J_2 criteria even if this is a side-effect of the main objective.

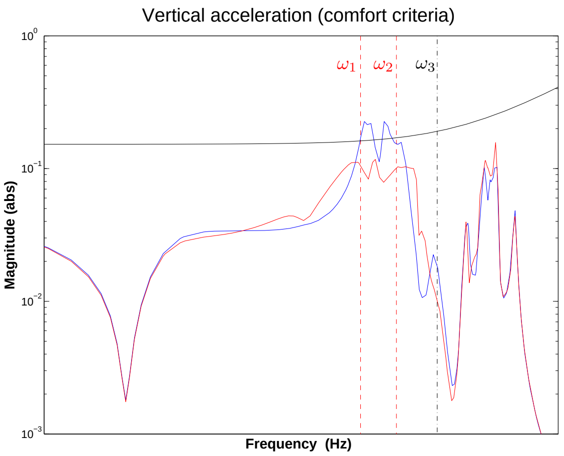

Fig. 6. Pilot acceleration response to an aerodynamic disturbance on the rear part of the A/C. Without anti-vibration law (solid blue), with anti-vibration control (solid red) and γ/W_{aw} weight filter (solid black).

Figures 6 illustrates the vibration attenuation achieved on $T_{w_a \to a_{pil}}$ applied on the reduced-order model (r = 50). Figure 7 gathers the frequency responses of $T_{w_a \to a_{pil}}$ (top frame) and $T_{u_a \to a_{cont}}$ (bottom frame) of the closed-loop system, including the full-order model (n 600). Without anti-vibration law (in blue), the transfer $T_{w_a \to a_{pil}}$ (top frames of Figure 7) is subject to an undesirable peak in the interval $^{(1)}$ = [ω_1 ω_2], which is reduced with a ratio around 50% with the anti-vibration control law (in red). Moreover, regarding the transfer $T_{u_a \to a_{cont}}$ (bottom frames of Figure 7), when the anti-vibration laws is added (in red), its magnitude remains in the envelope 20% of the nominal transfer (dotted blue lines) in the frequency interval $^{(2)}$ = [0 ω_c], avoiding degradation of nominal performances.

4. CONCLUSIONS AND PERSPECTIVES

This paper addresses the problem of designing an anti-vibration controller on a LTI large-scale BizJet generic model. The contributions are in two folds: *(i)* first, a successful application of modern (frequency-limited) H_2 dynamical model approximation techniques on a complex aeorelastic aircraft model, has been demonstrated.

Then, *(ii)* an application to an engineering test case (reduced model) using a structured H_∞ method generating low order controllers has been performed. The resulting anti-vibration control laws obtained thanks to the reduced model (Figure 6) have been then tested on the original large-scale high fidelity model (Figure 7) with a very small degradation.

This first step study encourages us to investigate further developments such as *(i)* the extension to multi-LTI models in order to include the uncertainties and variability of the system ($e.g.$ with respect to the mass, flight condition, actuators, ...) in a robust framework, *(ii)* the minimization of external perturbation effect ($e.g.$ atmospheric turbu-

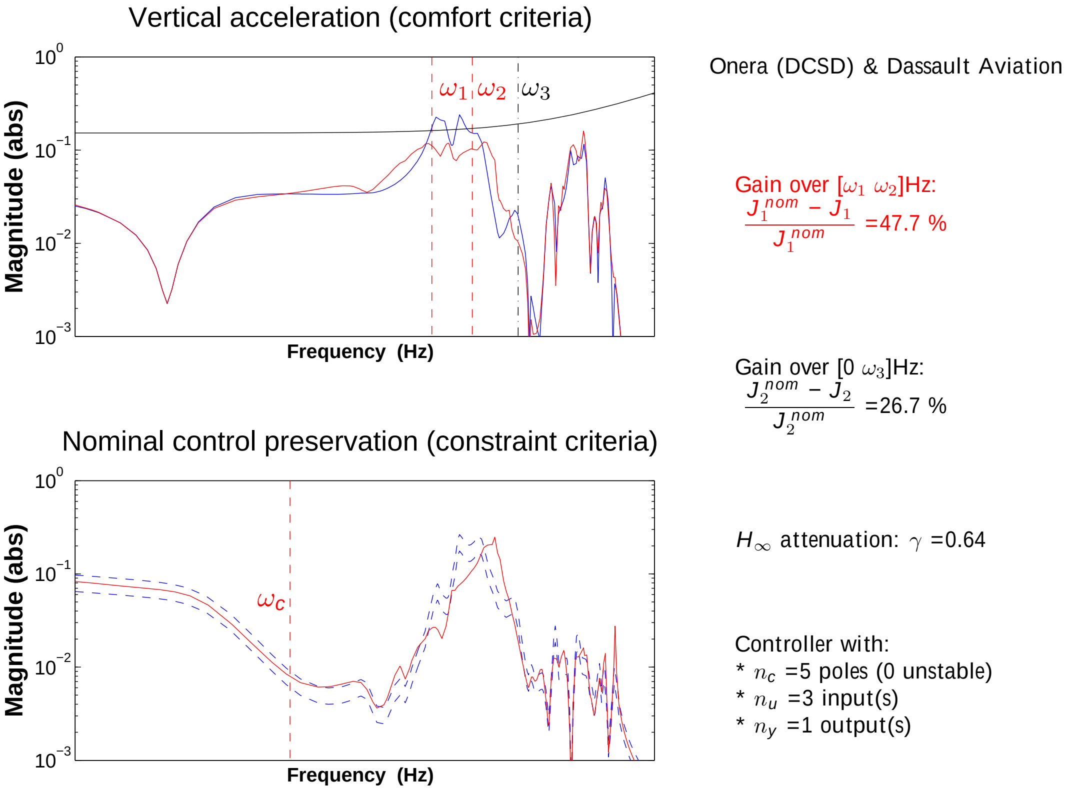

Fig. 7. Full order model simulation. Pilot acceleration response to an aerodynamic disturbance on the rear part of the A/C (top frame) and flight control nominal signal (bottom frame). Without anti-vibration law (solid blue), envelope 20% of the nominal transfer (dashed blue), with anti-vibration control (solid red) and γ/W_{aw} weight filter (solid black).

lence, sensors noise, ...), and *(iii)* the respect of constraints related to discretization and computing rate.

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